Excited Hadrons, Heavy Quarks and QCD thermodynamics* **

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We show how excited states in QCD can be profitably used to build up the Polyakov loop in the fundamental representation at temperatures below the hadron–quark-gluon crossover. The conditions under which a Hagedorn temperature for the Polyakov loop can be defined are analyzed.

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1. Introduction

The QCD equation of state can be derived from the partition function

$$Z(T) = \mathrm{Tr} e^{-H/T} = \sum_n g_n e^{-E_n/T} . \tag{1}$$

In lattice QCD with 2+1 flavours $Z(T)$ has been evaluated by the HotQCD [1] and Wuppertal-Budapest [2] collaborations producing different results for the trace anomaly at temperatures above $T = 200\mathrm{MeV}$, already beyond the hadron–quark-gluon crossover [3]. On the other hand, Quark-Hadron

duality at finite temperature requires that for confined states Z should be determined from *all* stable hadron states such as those in the PDG booklet [4]. This is the idea behind the Hadron Resonance Gas (HRG), a multi-component gas of non-interacting massive stable and point-like particles [5] which has historically arbitrated the discrepancies between different lattice groups [6–8]. Remarkably, the disagreement still persists beyond the expected range of validity of the HRG model (see e.g. Fig. 1, right).

The special role played by the HRG does not make it a theorem and corrections to it are not completely clear as PDG hadrons are composite, have finite size and width. On the lattice, the validity of the HRG has been checked in the strong coupling limit and for heavy quarks to lowest orders [9]. In the usual large N_c-limit (see Ref. [10] for a review and references therein) where hadrons become stable resonances, $\Gamma/M = \mathcal{O}(1/N_c)$, the mesons give a finite contribution as their mass and degeneracy are finite whereas baryons would provide a vanishing contribution. The half-width rule [11] applied to PDG resonances [4] provides compatible uncertainties with current lattice calculations [2].

To saturate the partition function, Eq. (1) with light or heavy quarks a large number of highly excited states is needed so relativistic corrections are important. Here, we will use the MIT Bag model [12] and the Relativized Quark Model (RQM) of Refs. [13, 14] which treats hadrons as extended bound states rather than resonances.

2. Trace anomaly and light quarks

The trace anomaly measures departures from scale invariance and reads

$$\mathcal{A}(T) \equiv \frac{\epsilon - 3p}{T^4} = T\frac{\partial}{\partial T}\left(\frac{p}{T^4}\right), \qquad (2)$$

after using standard thermodynamics relations for the energy density $\epsilon = E/V$ and pressure $p = -T\log Z/V$. For the HRG model we have

$$\mathcal{A}(T) = \frac{1}{T^4}\int_0^\infty dM \frac{dn(M)}{dM}\int \frac{d^3k}{(2\pi)^3}\frac{(E_k - \vec{k}\cdot\nabla_k E_k)}{e^{E_k/T} \pm 1}, \qquad (3)$$

where $E_k = \sqrt{k^2 + M^2}$ and $\pm$ corresponds to Fermions/Bosons and

$$n(M) = \sum_\alpha g_\alpha \Theta(M - M_\alpha), \qquad (4)$$

is the cumulative number (Θ is the step function). Hagedorn proposed that the cumulative number of hadrons in QCD is approximately and asymptotically given by $n(M) = A\,e^{M/T_H}$ where A is a constant and T_H is the so
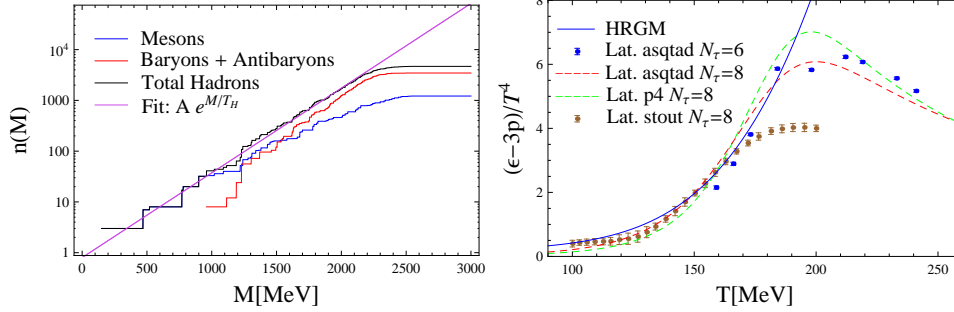
Fig. 1. Left: Cumulative number n as a function of the hadron mass M (in MeV) with u, d and s quarks, computed in the RQM [13, 14] and compared to a fit $n(M) = Ae^{M/T_H}$. Right: Trace anomaly $(\epsilon - 3p)/T^4$ as a function of temperature (in MeV). We compare lattice data for asqtad and p4 [15] (after temperature downshift of $T_0 = 15\,\mathrm{MeV}$) and stout [16] actions, with the HRGM computed with the RQM spectrum with u, d and s quarks from Refs. [13, 14].
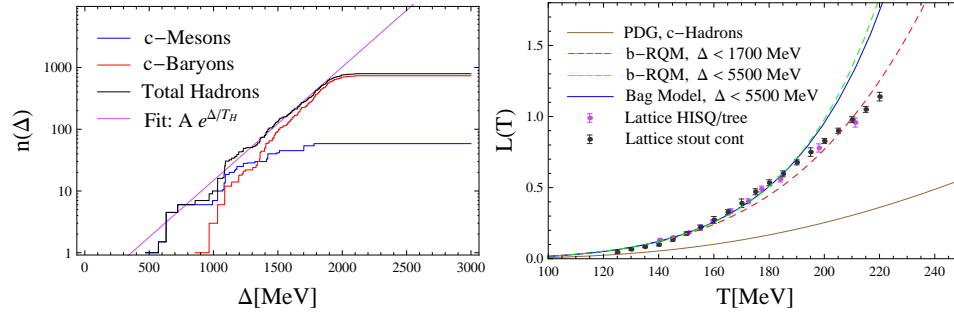

Fig. 2. Left: Cumulative number n as a function of the c-quark mass subtracted hadron mass $\Delta = M - m_c$ (in MeV) with u, d and s quarks, computed in the RQM [13, 14] and compared to a fit $n(\Delta) = Ae^{\Delta/T_H}$. Right: Polyakov loop as a function of temperature (in MeV). Lattice data from [17] for the HISQ/tree action and [16] for the continuum extrapolated stout result. We compare lowest-lying charmed hadrons from PDG [4], the RQM spectrum with one b quark and a cut-off $\Delta < 1700\,\mathrm{MeV}$ (red line), and $\Delta < 5500\,\mathrm{MeV}$ and the MIT bag model ($m_h \to \infty$) with cut-off $\Delta < 5500\,\mathrm{MeV}$ is shown as a solid (blue) line [18].

called Hagedorn temperature. We show results in Fig. 1 both for $n(M)$ and $\mathcal{A}(T)$ fitted with $A = 0.80$ and $T_H = 260\mathrm{MeV}$ and also showing the good performance of the HRG below $T = 180\mathrm{MeV}$ when the RQM is used.

3. Polyakov loop and heavy quarks

The Polyakov loop is a purely gluonic operator, which in gluodynamics becomes a true order parameter as it signals the breaking of the center symmetry and deconfinement. Unlike the trace anomaly, there is lattice consensus on this observable [16, 17] so its analysis may be more credible. We have shown that in QCD [18, 19] and in chiral quark models [20] a hadronic representation exists and is given by (A_0 is the gluon field)

$$L_T = \langle \mathrm{tr}_c \mathsf{P} e^{i \int_0^{1/T} A_0 \, dx_0} \rangle = \frac{1}{2} \int d\Delta \frac{\partial n(\Delta)}{\partial \Delta} e^{-\Delta/T} \,, \tag{5}$$

where the cumulative number reads now

$$n(\Delta) = \sum_\alpha g_{h\alpha} \Theta(\Delta - \Delta_{\alpha,h}) \,, \tag{6}$$

where $g_{h\alpha}$ are the degeneracies and $\Delta_{h\alpha} = M_{h\alpha} - m_h$ are the masses of hadrons with exactly one heavy quark (the mass of the heavy quark itself m_h being subtracted).

The result with u, d and s quarks, computed in the RQM [13, 14] when the large but finite charmed quark mass, $m_h = m_c$ (using b-quarks does not change much), is taken is presented in Fig. 2. We have checked that results are not very sensitive to use bottom quarks instead. A fit $n(\Delta) = A e^{\Delta/T_H}$ to the total contribution produces $A = 0.216, 0.209$ and $T_H = 236, 207$MeV for single-charmed, bottom hadrons for the range $1\mathrm{GeV} \leq \Delta \leq 1.8\mathrm{GeV}$. The results from PDG and RQM are multiplied by a factor $L(T) \to e^{C/T} L(T)$, with $C = 25\,\mathrm{MeV}$, which comes from an arbitrariness in the renormalization. The sum rule has been implemented on the lattice recently [21].

4. The non-overlapping condition

In the quantum virial expansion [22], the excluded volume corrections come from repulsive interactions whereas resonance contributions stem from attractive interactions. A good example is $\pi\pi$ scattering where one has attractive and resonating states in the isospin $I = 0, 1$ corresponding to the σ and ρ resonances whereas one has a repulsive core in the $I = 2$ exotic channel [23, 24] providing a measure of the finite pion size. In contrast, the HRG assumes point-like elementary particles. However, in the narrow width limit resonances also have a finite size as they become bound states. Clearly, when hadrons overlap, the HRG model becomes invalid since the Pauli principle blocks many states allowed by colour neutrality. The non-overlapping condition corresponds to the inequality for the Co-volume

$$\mathrm{CoV} \equiv \sum_i V_i N_i \leq V \,, \qquad \sum_i V_i \int \frac{d^3 p}{(2\pi)^3} \frac{g_i}{e^{E_i(p)/T} \pm 1} \leq 1 \,. \tag{7}$$
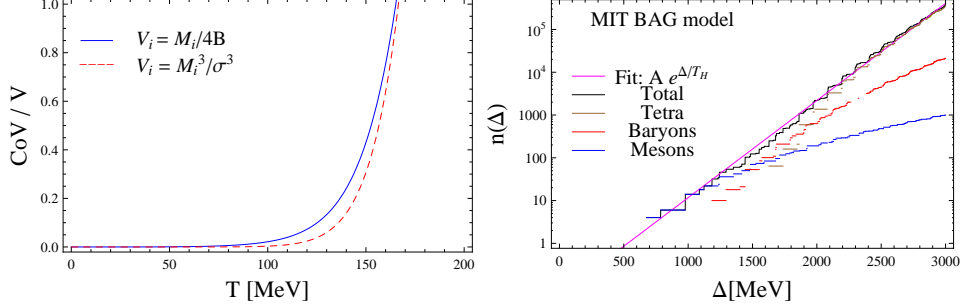
Fig. 3. Left: Non-ovelapping condition as a function of temperature. For the hadron volume we use $V_i = M_i/4B$ with $B = (0.166\text{GeV})^4$ for the MIT bag volume (blue) and also $V_i = M_i^3/\sigma^3$ with $\sqrt{\sigma} = 0.42\text{GeV}$ (red). Right: Cumulative number $n(\Delta)$ in the MIT Bag model. We include contributions from $Q\bar{q}$, Qqq and $Q\bar{q}q\bar{q}$.

The hadron size can be estimated from the MIT bag model where one has [12] $V_i = M_i/(4B)$. In the RQM [13, 14] one might compute the size directly from the m.s.r. of the wave functions. A meson model of the form $M = 2p + \sigma r$ with $p \sim 1/r$ yields after minimizing $V = 4\pi r^3/3 \sim M^3/\sigma^3$. In Fig. 3 we see that for $T = 160 - 170\text{MeV}$ hadrons overlap and the HRG departs from the lattice QCD results (see Fig. 1, left).

5. Hagedorn and the bootstrap

The cumulative numbers computed in the RQM exhibit lower thresholds for mesons than baryons but the latter dominate due to the larger multiplicity of qqq than $q\bar{q}$ states, and eventually an exponential growth characterized by a Hagedorn temperature seems to set in (Figs. 1 and 2). Due to the finite number of degrees of freedom both mesons and baryons have a power-like behaviour for large masses $M \gg \sqrt{\sigma}$ producing a dimensional estimate $n_{\bar{q}q}(M) \sim M^6/\sigma^3$ and $n_{qqq}(M) \sim M^{12}/\sigma^6$ featuring the available phase space. An intriguing issue is under what conditions this exponential growth goes on high up in the spectrum as initially speculated by Hagedorn [5]. In Fig. 3 (right) we show $n(\Delta) = n_{Q\bar{q}}(\Delta) + n_{Qqqq}(\Delta) + n_{Q\bar{q}q\bar{q}}(\Delta) + \ldots$ in the MIT Bag model including also the exotic tetraquark $Qq\bar{q}q$ states as independent hadronic states. The fit yields $T_H \sim 191\text{MeV}$, complying with the bootstrap mechanism proposed long ago [25, 26]. Since some of the tetraquark states are of molecular nature, it is unclear if they should be incorporated in the cumulative number. This is related to the completeness or redundancy of hadronic states, particularly in the PDG as noted in [11].

6. Conclusions

The thermodynamical analysis of the hadronic spectrum has an increasing lack of energy resolution for increasing temperatures and a slowly converging pattern requiring many excited states. On the other hand, lattice calculations become difficult at very low temperatures where the main energy gaps are found. While this explains why the HRG model works well as function of temperature it is not obvious how to systematically compute deviations from this simple limit.

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